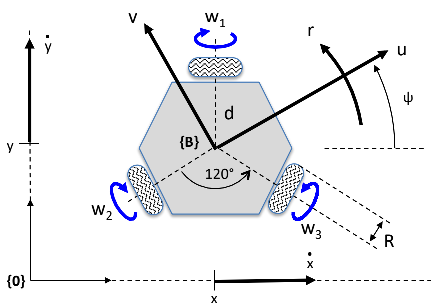
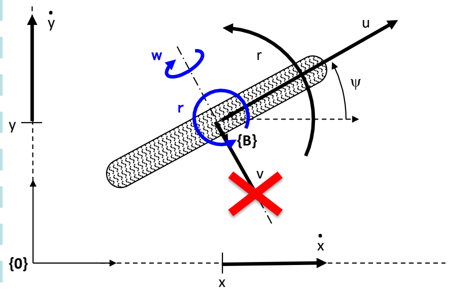
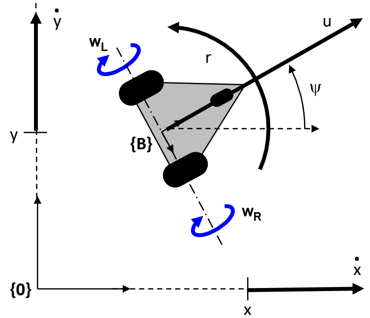
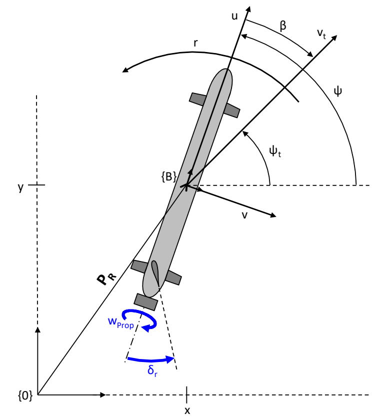
V – Guidance

Kinematics : the Wheel, the Unicycle and the AUV









$$\begin{bmatrix}\dot{x}\\ \dot{y}\\ \dot{\psi}\end{bmatrix}=\begin{bmatrix}\cos\psi & -\sin\psi & 0\\ \sin\psi & \cos\psi & 0\\ 0 & 0 & 1\end{bmatrix}\cdot\begin{bmatrix}u\\ v\\ r\end{bmatrix}$$

Linear approach OK

$$v=\dot{x}\cdot\cos\psi-\dot{y}\cdot\sin\psi=0$$

$$\begin{bmatrix}\dot{x}\\ \dot{y}\\ \dot{\psi}\end{bmatrix}=\begin{bmatrix}\cos\psi & 0\\ \sin\psi & 0\\ 0 & 1\end{bmatrix}\cdot\begin{bmatrix}u\\ r\end{bmatrix}$$

Linear approach is inapplicable

$$\begin{bmatrix}\dot{x}\\ \dot{y}\\ \dot{\psi}\end{bmatrix}=\begin{bmatrix}\cos\psi & -\sin\psi & 0\\ \sin\psi & \cos\psi & 0\\ 0 & 0 & 1\end{bmatrix}\cdot\begin{bmatrix}u\\ v\\ r\end{bmatrix}$$

$$\begin{bmatrix}\dot{x}\\ \dot{y}\\ \dot{\psi}\end{bmatrix}=\begin{bmatrix}\cos\psi_t & 0\\ \sin\psi_t & 0\\ 0 & 1\end{bmatrix}\cdot\begin{bmatrix}v_t\\ r+\dot{\beta}\end{bmatrix}$$

$$F_u=m_u\cdot\dot{u}+d_u$$

$$0=m_v\cdot\dot{v}+m_{ur}\cdot u\cdot r+d_v$$

$$\Gamma=m_r\cdot\dot{r}+d_r$$

$$\underbrace{\begin{bmatrix}w_1\\ w_2\\ w_3\end{bmatrix}}_{\chi}=\frac{-1}{R}\cdot\underbrace{\begin{bmatrix}-\sin\alpha_1 & \cos\alpha_1 & d\\ -\sin\alpha_2 & \cos\alpha_2 & d\\ -\sin\alpha_3 & \cos\alpha_3 & d\end{bmatrix}}_{\mathbf{M}_A^{-1}}\cdot\underbrace{\begin{bmatrix}u\\ v\\ r\end{bmatrix}}_{\upsilon}$$

$$\begin{bmatrix}u\\ r\end{bmatrix}=\begin{bmatrix}R & 0\\ 0 & 1\end{bmatrix}\cdot\begin{bmatrix}w\\ r\end{bmatrix}$$

$$\begin{bmatrix}u\\ r\end{bmatrix}=\begin{bmatrix}\frac{R}{2} & \frac{R}{2}\\ \frac{R}{2\cdot L} & \frac{-R}{2\cdot L}\end{bmatrix}\cdot\begin{bmatrix}w_l\\ w_r\end{bmatrix}$$

$$\mathbf{F}_B=\begin{bmatrix}F_u\\ \Gamma\end{bmatrix}=\mathbf{M}_A\cdot\mathbf{F}_m(w_{prop},\delta)$$

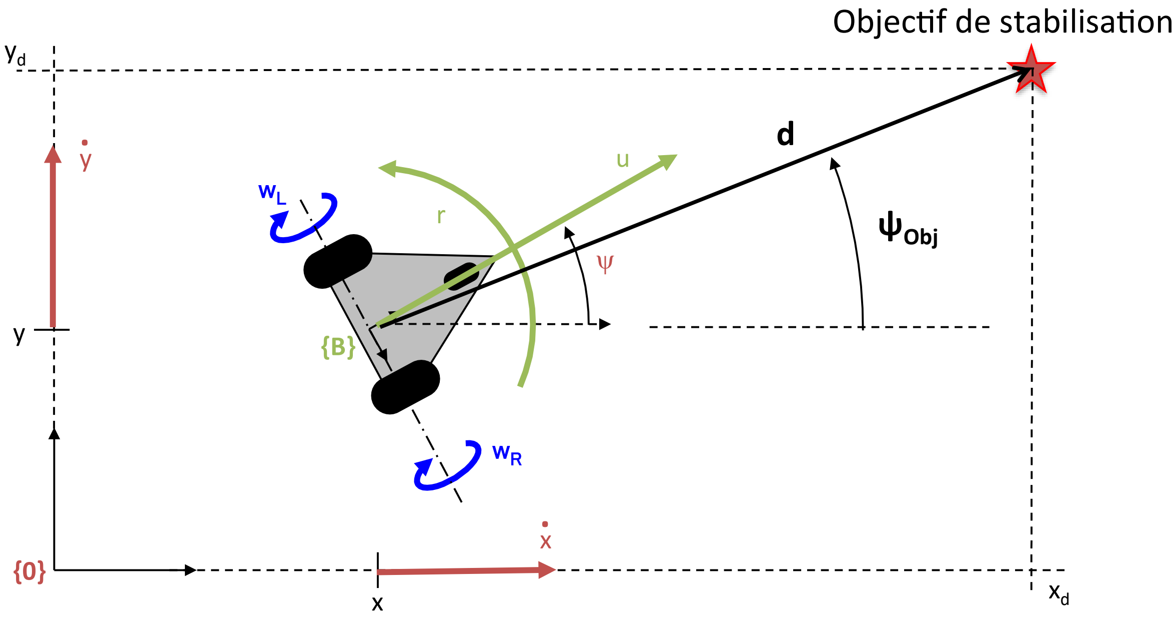
The transposition of Unicycle solution to AUV depends on the system shape. $\dfrac{m_{ur}}{m_v}<1$

Regulation objectives

- Stabilisation
  - Stabilize the vehicle at a given point, with given orientation, and maintain them.
- Trajectory tracking
  - Track a time-parameterized spatial reference
- Path Following
  - Converge to a precribed path, without any temporal specification

Commande

- Contrôler le mouvement du robot
  - Stabilisation d'un système non-holonome
    - Rejoindre un point, et s'y arrêter.



$$d \to 0$$

- Pointer l'objectif :

$$\psi \to \psi_d$$

- Avancer jusqu'à l'objectif :

$$u = f(d) / \lim_{d \to 0} u = 0$$

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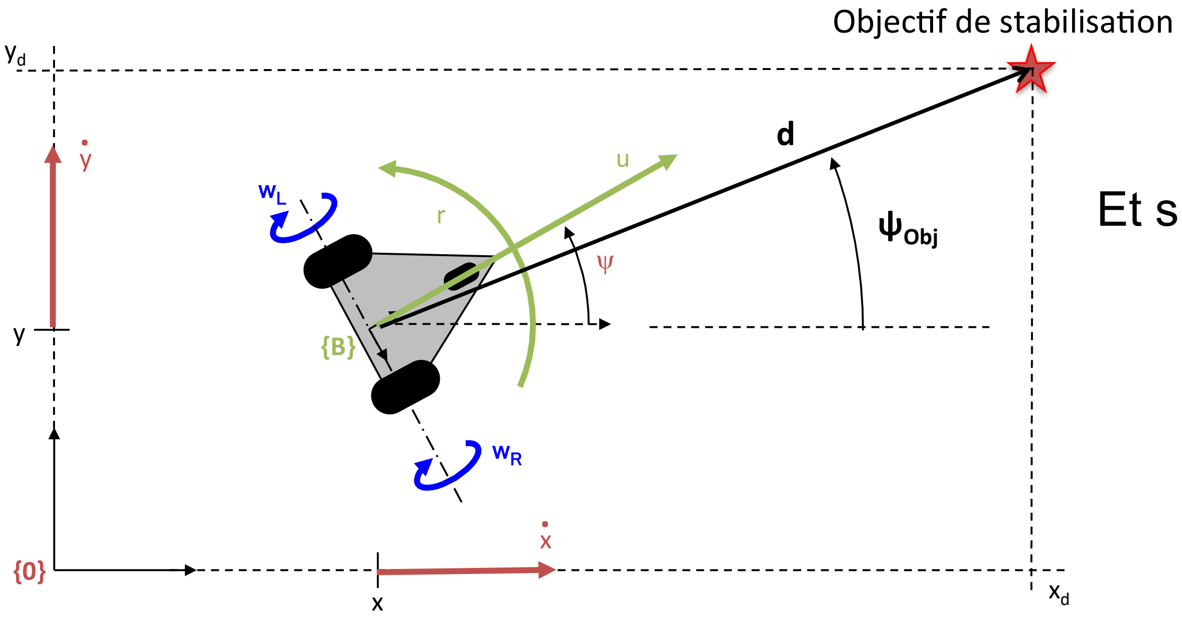

- Pointer l'objectif :

$$\tilde{\psi} = \psi_d - \psi \overset{?}{\rightarrow} 0$$

Et si on arrivait à faite en sorte que :

$$\dot{\tilde{\psi}} \overset{?}{=} -K_\psi \cdot \tilde{\psi}$$

$$\Rightarrow r = \dot{\psi} \overset{?}{=} \dot{\psi}_d + K_\psi \cdot \tilde{\psi}$$

- Avancer jusqu'à l'objectif :

$$u = f(d) / \lim_{d \to 0} u = 0$$

$$\text{ex: } u = d$$

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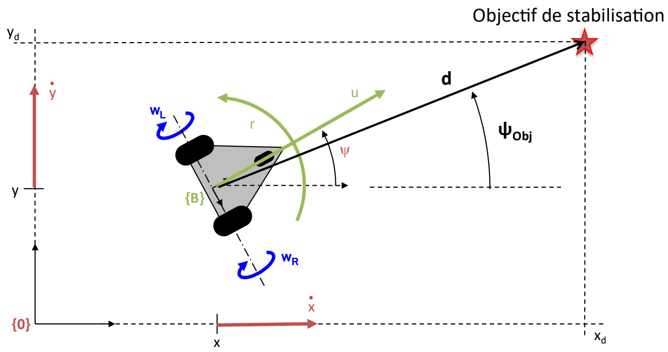
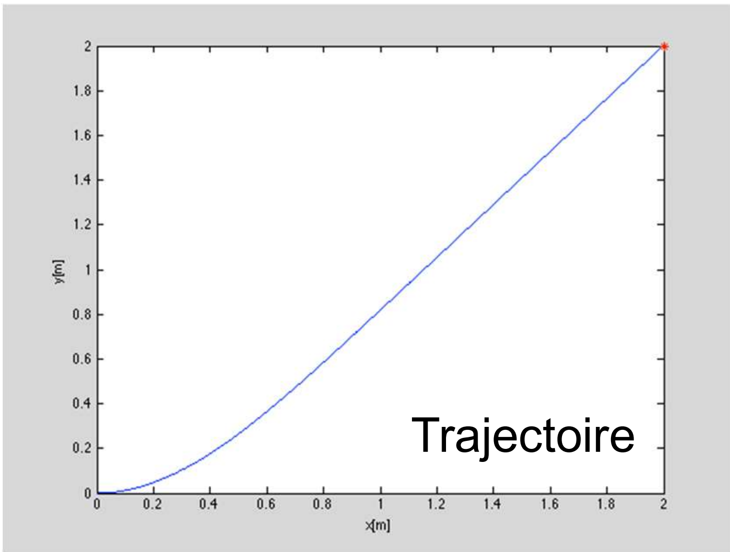


$$\begin{cases} u_c = d/10 \\ r_c = \dot{\psi}_d + K_\psi \cdot \tilde{\psi} \end{cases} \rightarrow \boxed{M_A^{-1}} \rightarrow \begin{cases} w_L \\ w_R \end{cases}$$



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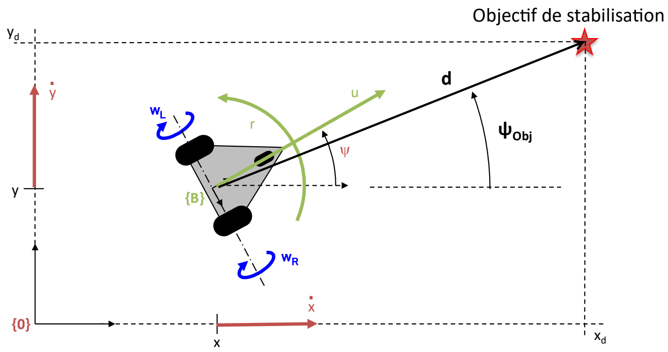

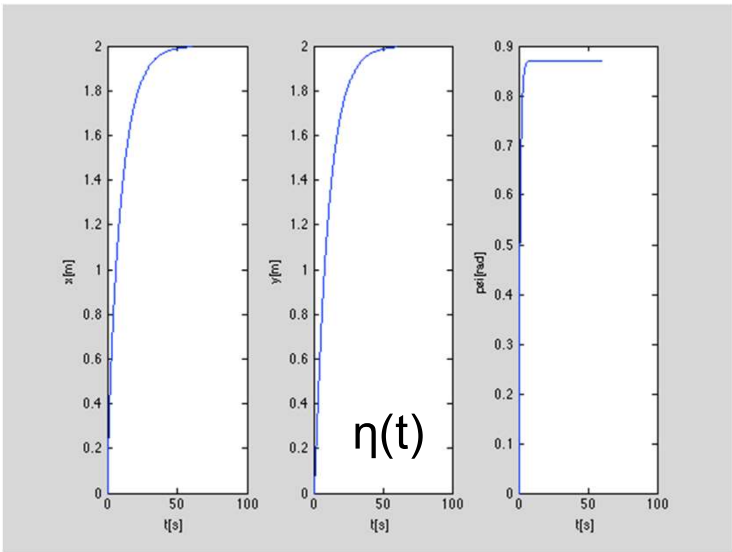

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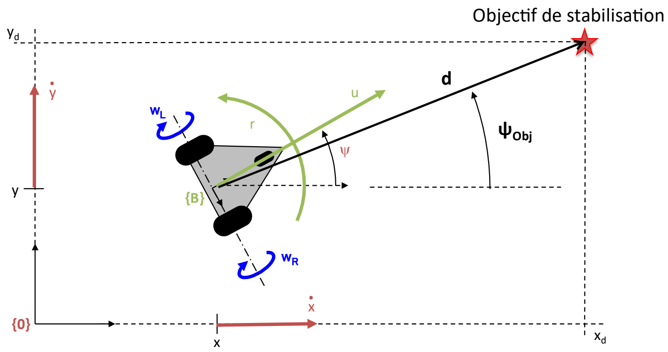

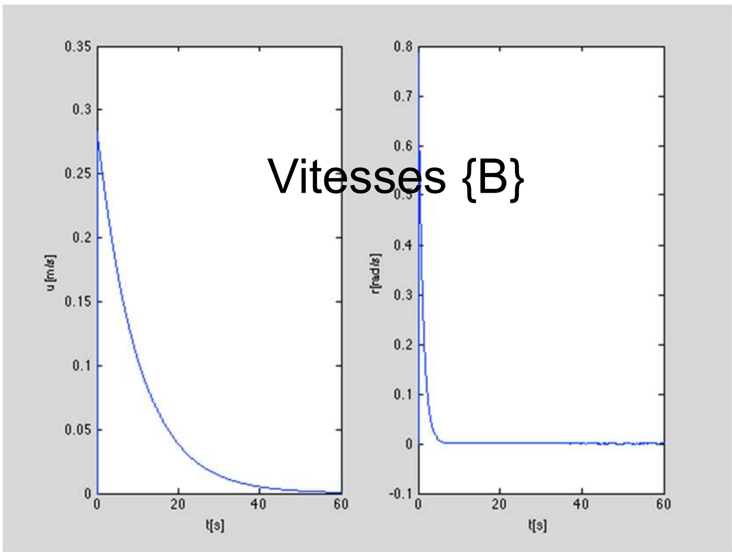

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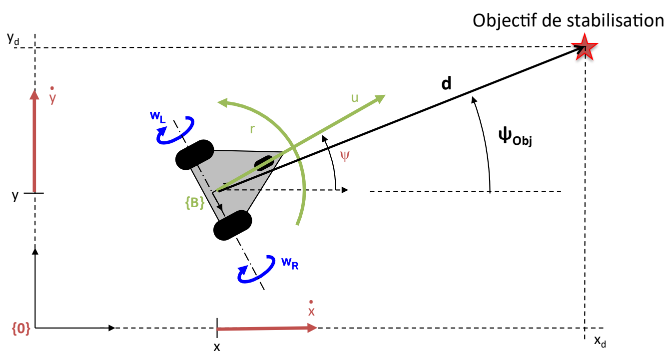

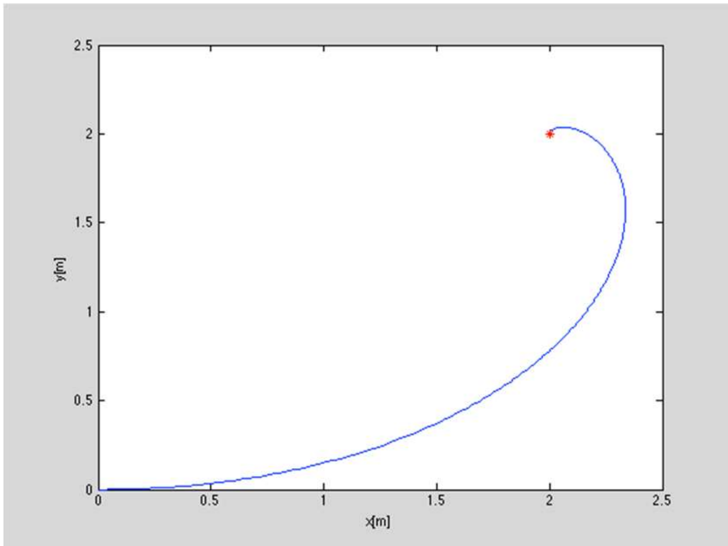

$$\left\{ \begin{array}{c} u_c = d \\ r_c = \dot{\psi}_d + K_\psi \cdot \tilde{\psi} \end{array} \right\} \rightarrow \boxed{M_A^{-1}} \rightarrow \left\{ \begin{array}{c} w_L \\ w_R \end{array} \right\}$$

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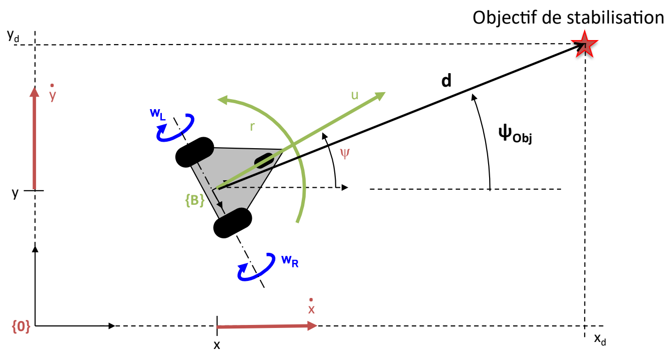

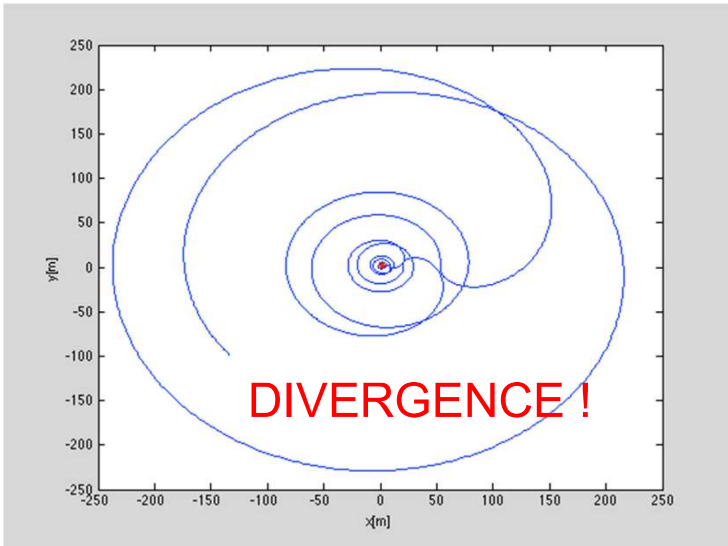

$$\begin{cases} u_c = 1.5 \cdot d \\ r_c = \dot{\psi}_d + K_\psi \cdot \tilde{\psi} \end{cases} \rightarrow \boxed{M_A^{-1}} \rightarrow \begin{cases} w_L \\ w_R \end{cases}$$

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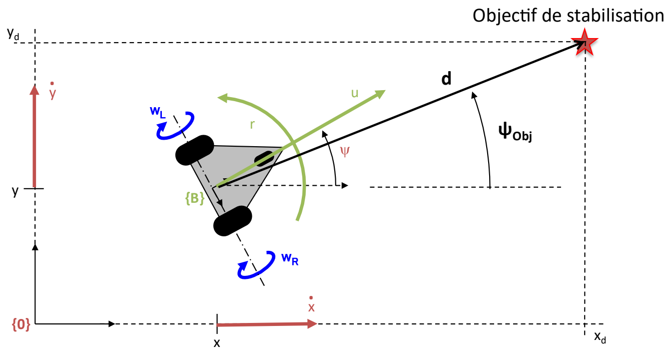

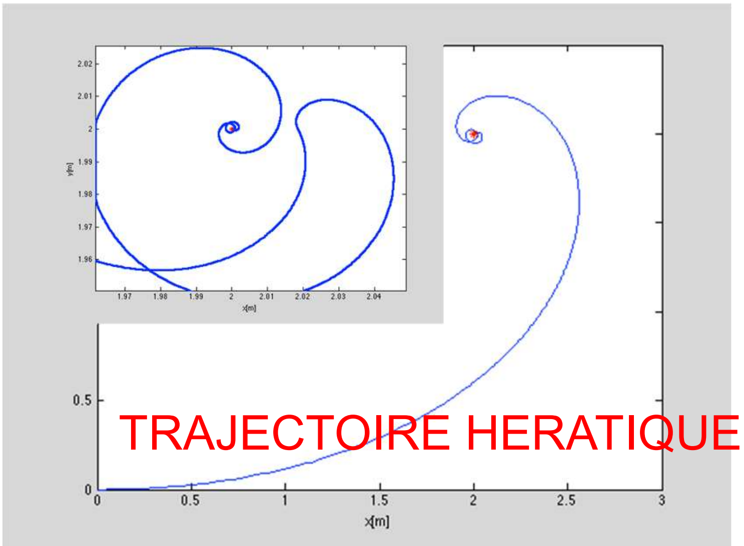

TRAJECTOIRE HERATIQUE !

$$\begin{Bmatrix} u_c = \hat{d} \\ r_c = \dot{\psi}_d + K_\psi \cdot \tilde{\psi} \end{Bmatrix} \rightarrow \boxed{M_A^{-1}} \rightarrow \begin{Bmatrix} w_L \\ w_R \end{Bmatrix}$$

$$\hat{d} = d \cdot (1 + 0.1 \cdot \text{rand}())$$

Commande

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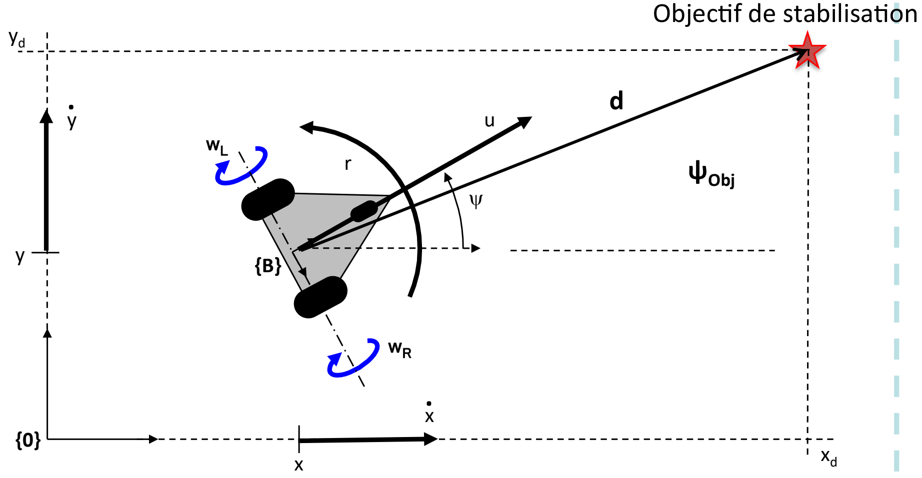
Il n'existe pas de loi continument dérivable qui puisse stabiliser un système non-holonome !!!
[BROCKETT 83]

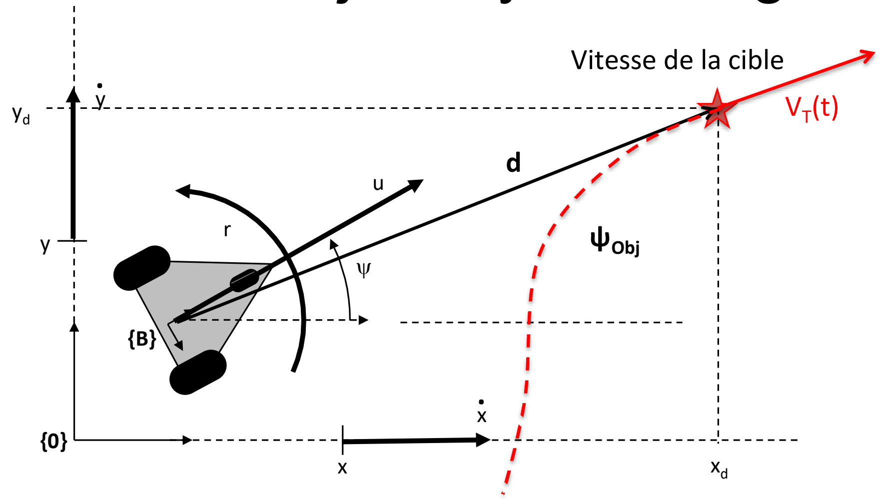
- L'objectif est singulier
- Une petite erreur provoque une grande manœuvre
- Le linéarisé n'est plus contrôlable sur l'objectif

2. Choose movement control type

Stabilisation

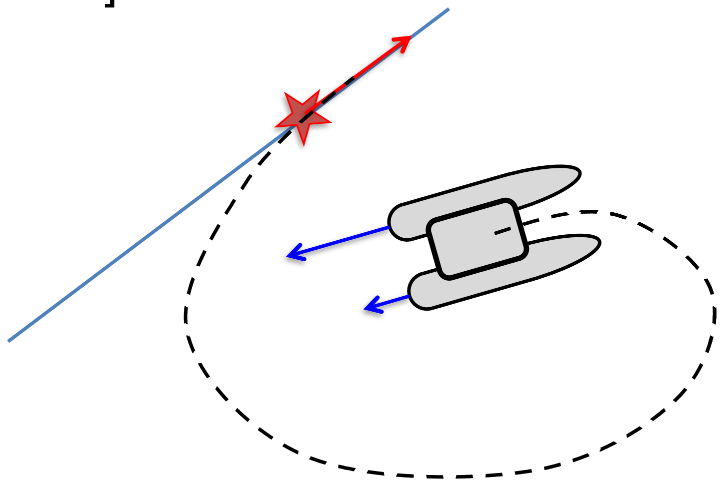


Trajectory Tracking



There is NO continuous control law that could stabilise a nonholonomic system [BROCKETT 83]

- Singular objective
- A small error induces a large manoeuvre
- The linearized model is not controllable
- Trajectory : Space & **Time** reference
- The vehicle may turn back in its attemp to be at a desired reference at a prescribed time



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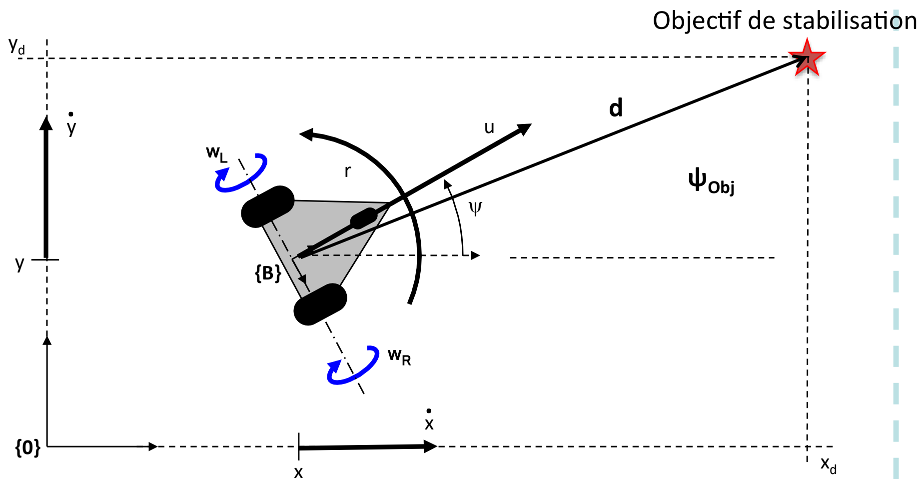

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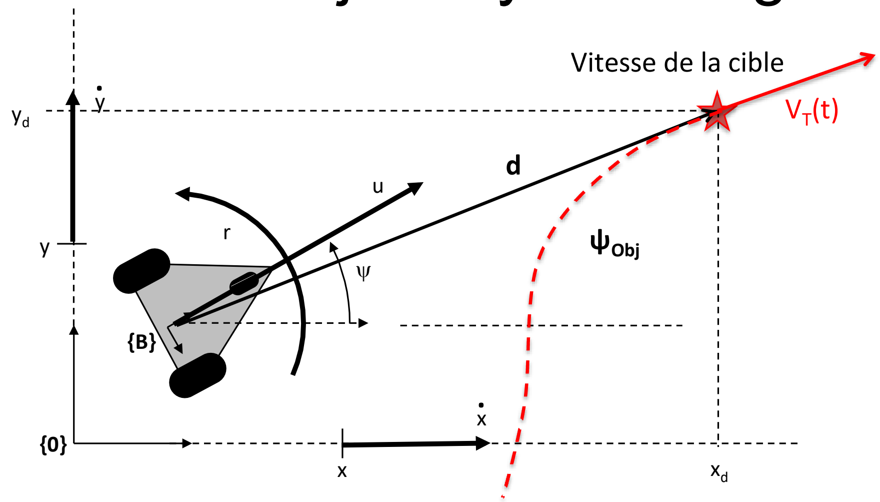

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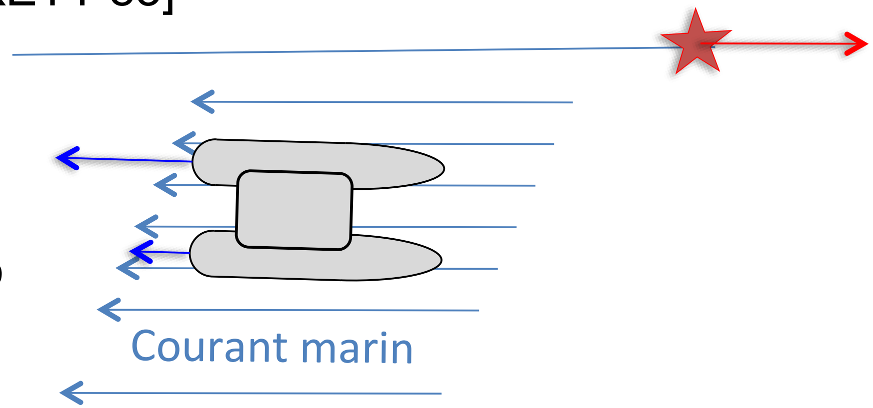
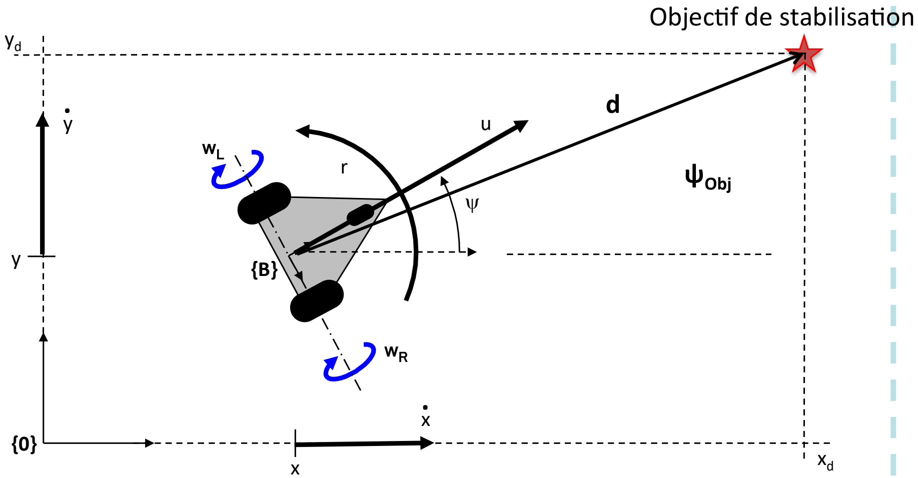
- Actuators may easily be pushed to saturation



Stabilisation



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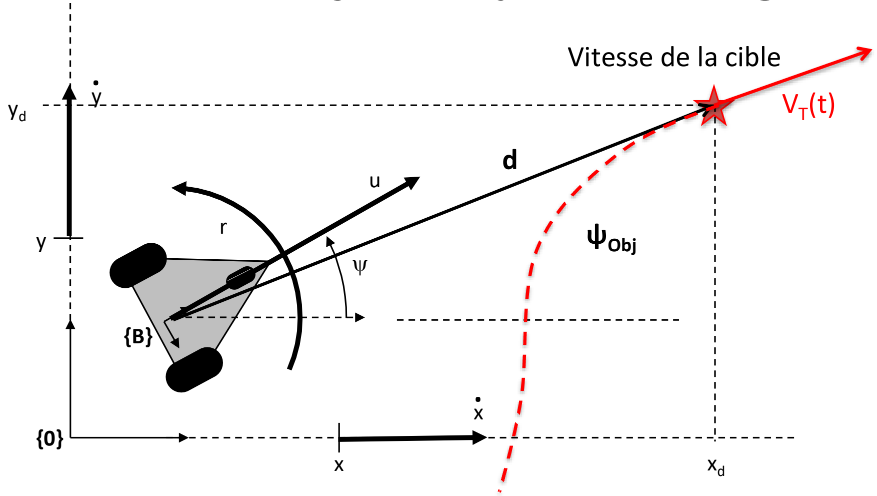

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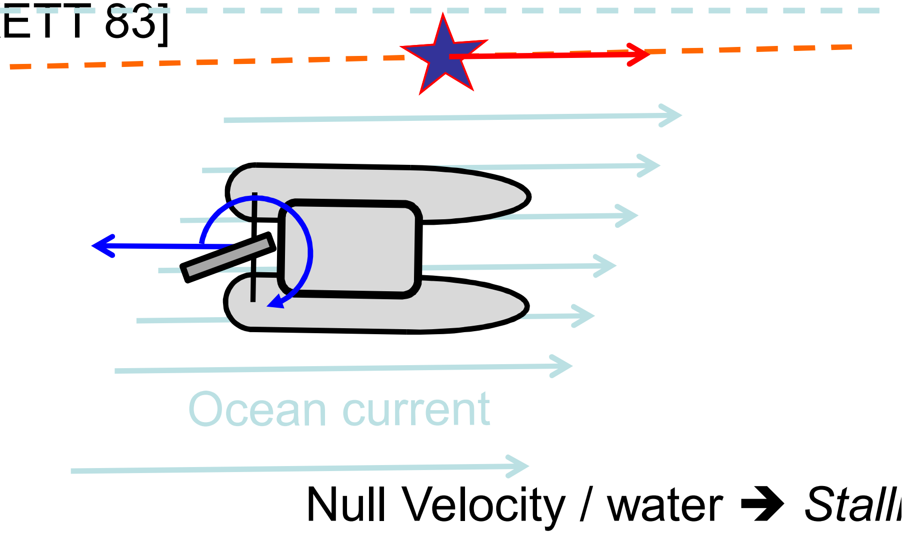
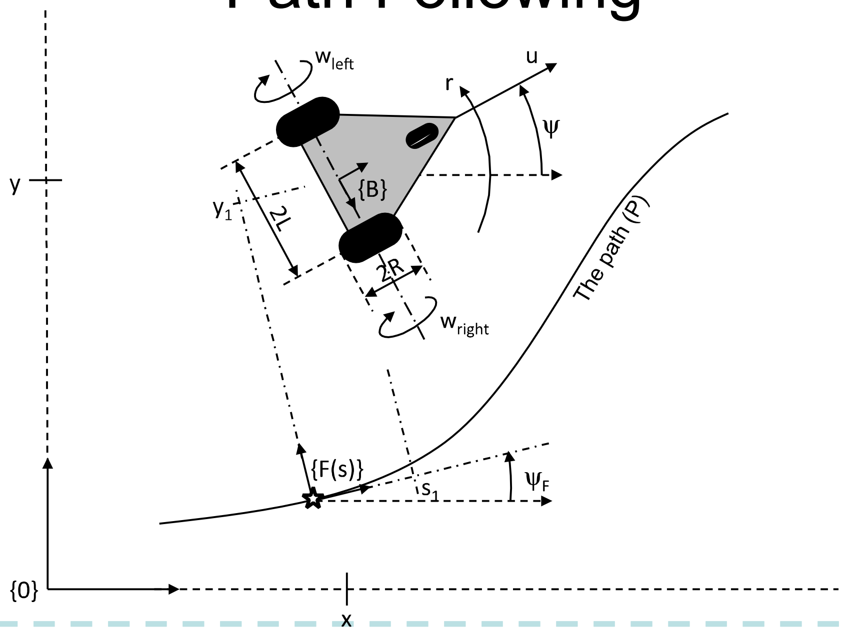
- Danger of **Stalling**



Path Following



- Decoupled control u and r
- Remove temporal constraint
- Autonomous system
- ‘smooth’ convergence to the path
- Keep control on actuator saturation
- Global and Uniform Asymptotic Converge
- Can be extended to RDV tracking
- Can be extended to formation keeping

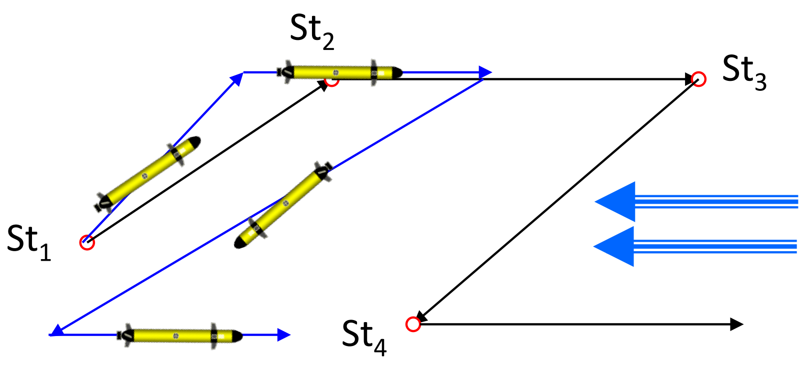
~~There is NO continuous control law that could stabilise a nonholonomic system [BROCKETT 83]~~

- ~~Singular objective~~
- ~~A small error induces a large manoeuvre~~
- The linearized model is not controllable
- Path Following : Space & **~~Time~~** reference
- ~~The vehicle may turn back in its attemp to be at a desired reference at a prescribed time~~
- ~~Actuators may easily be pushed to saturation~~
- ~~Danger of **Stalling**~~

3. Choose Guidance Strategy

Set-points
(heading – Duration)

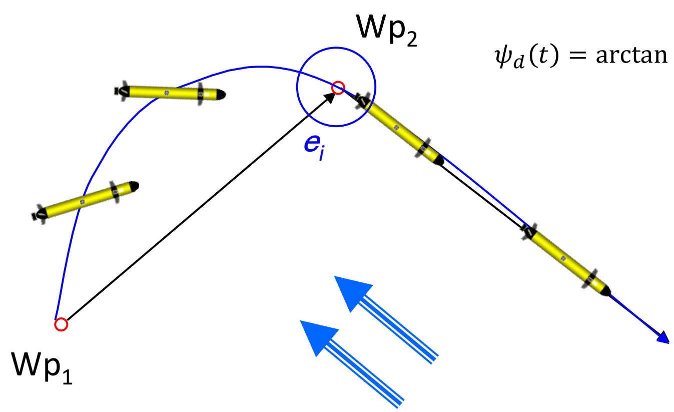
St_i	u_i	$\Psi_{d,i}$	t_i
St_1	1m/s	40°	30s
St_2	1m/s	90°	30s
St_3	1m/s	220°	30s
St_4	1m/s	40°	30s



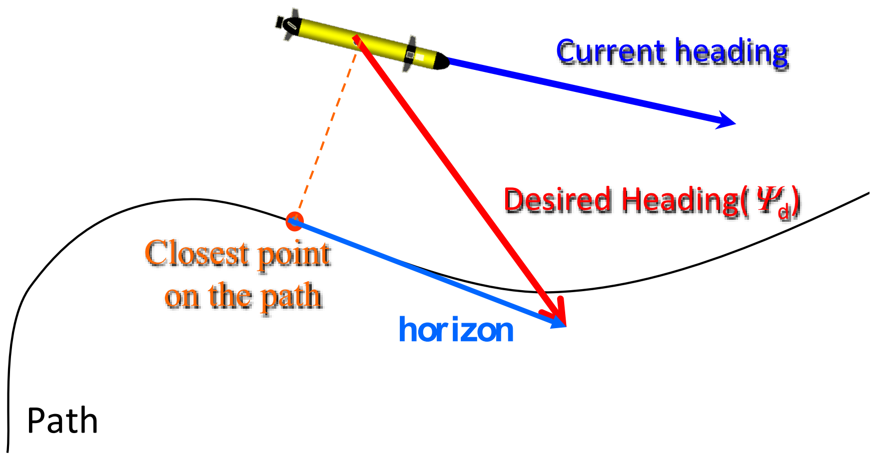
Waypoints
(geo-referenced)

Wp_i	u_i	x_i	y_i	e_i
Wp_1	1m/s	10m	20m	.1m
Wp_2	1m/s	0m	40m	.2m

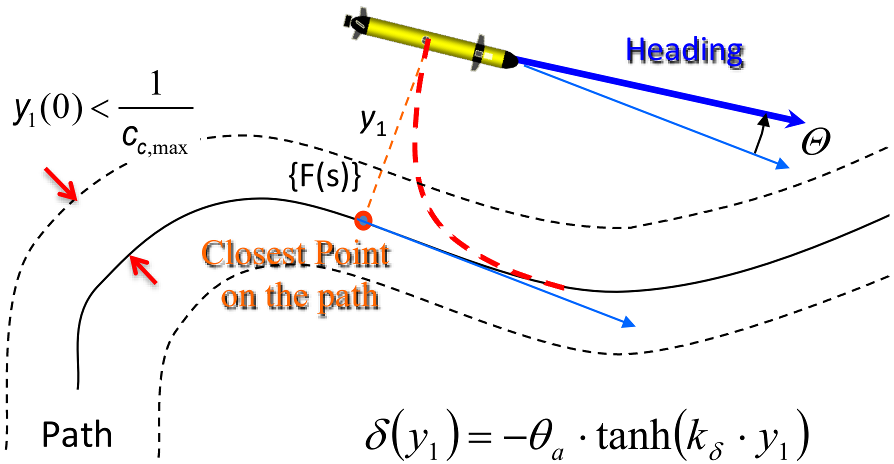
$$\psi_d(t) = \arctan\left(\frac{x_k - x(t)}{y_k - y(t)}\right)$$



Line of sight



Path Following



$$y_1(0) < \frac{1}{c_{c,\max}}$$

$$\delta(y_1) = -\theta_a \cdot \tanh(k_\delta \cdot y_1)$$